



RESEARCH ARTICLE

Experimental Estimation of Gravitational Acceleration Using Manual Free Fall Measurements

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ABSTRACT

Hand-timed free-fall experiments are a standard way for students to estimate the gravitational acceleration g , but it is often unclear whether the resulting errors come from air resistance or from the person operating the stopwatch. We dropped three everyday objects (an eraser in three orientations, a pencil sharpener, and a pencil in two orientations) from heights of 0.3 m and 1.0 m, with four hand-timed trials per configuration, and extracted g by inverting the exact solution of the quadratic-drag equation of motion. Air drag raises the extracted value of g by only 0.05–1.45% (mean 0.37%), a correction well described by the simple estimate $kh/3$, where k is the drag parameter. By contrast, the fall times of different configurations dropped from the same height differ by 25 ms (RMS), although drag predicts agreement within 0.7%. Using this observed scatter as the timing uncertainty, the inverse-variance weighted mean of all twelve configurations is $g = 10.29 \pm 0.42 \text{ m/s}^2$, 5% (1.2σ) above the local reference value $g_{\text{theo}} = 9.802 \text{ m/s}^2$. The measured times are, on average, $16 \pm 7 \text{ ms}$ shorter than those expected for free fall, consistent with a systematic early stop of the stopwatch; this offset explains why the 0.3 m drops overestimate g by 24% on average and the 1.0 m drops by 8%. We conclude that, for household objects dropped from about a metre, drag corrections are unnecessary and the accuracy of hand-timed free fall is limited by systematic timing bias, which is best addressed by measuring at several heights and fitting fall time against $\sqrt{2h}$.

Keywords: free fall, air drag, reaction time, systematic error, measurement uncertainty, physics education

1. INTRODUCTION

Measuring the acceleration due to gravity by timing a falling object is one of the first quantitative experiments many students perform. With a hand-operated stopwatch and ordinary objects the result usually differs noticeably from the accepted value, and students (and teachers) are left to guess why. Two explanations are commonly offered: air resistance, which slows the object, and the reaction time of the person operating the stopwatch. They push the result in opposite directions—drag lengthens the fall and lowers the apparent g , whereas stopping the watch early shortens it and raises the apparent g —so it is worth knowing which one actually dominates.

The reference value at the experimental site follows from the International Gravity Formula based on the Geodetic Reference System 1980 (GRS 80) [1], with a free-air correction for height z above sea level:

$$g_{\text{theo}} = g_0(1 + \gamma \sin^2 \varphi - \gamma' \sin^2 2\varphi) - k_h z \quad (1)$$

where $g_0 = 9.780327 \text{ m/s}^2$, $\gamma = 0.0053024$, $\gamma' = 0.0000058$ and $k_h = 3.086 \times 10^{-6} \text{ s}^{-2}$. For Gebze, Kocaeli ($\varphi \approx 40.8^\circ \text{ N}$, $z \approx 190 \text{ m}$) this gives $g_{\text{theo}} = 9.802 \text{ m/s}^2$. Local geological anomalies change this value by far less than the uncertainties discussed here.

This paper makes three contributions. First, it quantifies the drag correction exactly and shows that, for typical classroom objects and heights, it can be estimated with a one-line formula. Second, it separates the *repeatability* of hand timing (within a set of trials) from its *accuracy* (between configurations that should give identical times), and shows that the latter is much worse. Third, it shows that

the residual bias is consistent with a systematic early stop of the stopwatch and explains how a multi-height fit can remove it.

2. RELATED WORKS

Precise determinations of g rely on automated timing and controlled conditions. Absolute metrological measurements use free-fall or reversible-pendulum apparatus in vacuum or with optical tracking [7], [8]. In education, low-cost automated methods achieve millisecond-level timing with microcontrollers [2], smartphone sensors [3], [22], high-speed video [6], or pendulum experiments [9]. The effect of air resistance on falling objects has been studied in undergraduate laboratories using spheres, balls and balloons [4], [14], [16], [17].

Manual timing remains common in schools because it needs no equipment, and inquiry-based laboratories value exactly this kind of hands-on work [10], [21]. Human reaction times are well documented [11], [12], but a mean reaction time is not the same as the error in a timed interval: if the operator reacts with similar delays at start and stop, the delays partly cancel, and what remains is a mixture of random scatter and a systematic offset. Separating these two components with the data a student can realistically collect is the focus of this work.

3. METHODOLOGY

3.1. Objects, Configurations and Procedure

Three objects were used: a rectangular eraser dropped on each of its three faces (largest, medium, smallest), a pencil sharpener dropped on its flattest face, and a pencil dropped

in two orientations. Each of the six configurations was dropped from $h = 0.3$ m and $h = 1.0$ m, giving 12 configurations labelled ET01–ET06 (eraser), PST01–PST02 (sharpener) and PT01–PT04 (pencil). For each configuration $N = 4$ trials were timed with a stopwatch application on a laptop, started at release and stopped at impact by the same operator. Heights were marked on a vertical surface. Trials were performed in a closed room with ventilation off.

The projected areas A , masses m and drag coefficients C_d are listed in Table 1. The objects were not weighed; nominal masses typical of such objects were assigned, and C_d values for flat plates and short cylinders were taken from standard references [13], [15], [18]–[20]. Section 4.1 shows that these choices have no practical effect on the results.

Air density was calculated from the ideal gas law, $\rho = P_0 M_{\text{air}} / (RT)$, with $P_0 = 1.014 \times 10^5$ Pa (digital barometer), $T = 300$ K (infrared thermometer) and $M_{\text{air}} = 0.029$ kg/mol, giving $\rho = 1.179$ kg/m³.

Table 1: Object parameters, drag parameter k (Eq. 3) and terminal velocity v_t

Object, face	A (cm ²)	m (g)	C_d	k (m ⁻¹)	v_t (m/s)
Eraser, smallest	2.00	12.8	1.17	0.0108	30
Eraser, medium	4.00	12.8	1.17	0.0216	21
Eraser, largest	8.00	12.8	1.17	0.0431	15
Sharpener, planar	1.50	5.0	0.82	0.0145	26
Pencil, largest	1.40	12.0	1.20	0.0083	34
Pencil, smallest	0.785	12.0	1.20	0.0046	46

3.2. Equation of Motion with Quadratic Drag

With the downward direction positive and the object released from rest, Newton's second law with quadratic drag reads

$$m \frac{dv}{dt} = mg - \frac{1}{2} C_d \rho A v^2 \Rightarrow \frac{dv}{dt} = g - kv^2 \quad (2)$$

$$k = \frac{C_d \rho A}{2m}, \quad v_t = \sqrt{\frac{g}{k}} \quad (3)$$

The quadratic law is appropriate because the Reynolds numbers are of order 10^3 – 10^4 (speeds of 2–5 m/s, sizes of 1–3 cm). Integrating twice gives

$$v(t) = v_t \tanh\left(\frac{gt}{v_t}\right), \quad h(t) = \frac{v_t^2}{g} \ln \cosh\left(\frac{gt}{v_t}\right) \quad (4)$$

For each configuration, g_{exp} was obtained by solving Eq. (4) numerically for g given h and the measured fall time. We assume the objects did not tumble; visual observation supported this, although small rotations cannot be excluded.

Because the objects fall only a small fraction of the distance needed to approach v_t , expanding $\ln \cosh x \approx x^2/2 - x^4/12$ gives a useful closed form:

$$h \approx \frac{1}{2} g t^2 \left(1 - \frac{kgt^2}{6}\right) \Rightarrow \Delta_{\text{drag}} \approx \frac{kh}{3} \quad (5)$$

where $\Delta_{\text{drag}} = (g_{\text{drag}} - g_{\text{ideal}})/g_{\text{ideal}}$, $g_{\text{ideal}} = 2h/t^2$ is the vacuum estimate and g_{drag} is the solution of Eq. (4). Δ_{drag} is the

quantity that matters for the measurement: it is the amount by which ignoring drag biases g .

3.3. Statistics and Uncertainty

For each configuration we report the mean $\bar{g}$ of the four trial values, the sample standard deviation S and the standard error $SE = S/\sqrt{N}$. The signed relative deviation is

$$\delta g_{\text{rel}} = \frac{g_{\text{exp}} - g_{\text{theo}}}{g_{\text{theo}}} \times 100\% \quad (6)$$

A timing uncertainty σ_t propagates to $g \propto t^{-2}$ as

$$\sigma_g \approx \frac{2g}{t} \sigma_t \quad (7)$$

We compared two choices of σ_t : a nominal value of 50 ms based on reaction-time literature [11], [12], and an empirical value estimated from the data themselves (Section 4.2). Configurations were combined with the inverse-variance weighted mean $g_w = \sum w_i g_i / \sum w_i$, $w_i = \sigma_{g,i}^{-2}$, with uncertainty $(\sum w_i)^{-1/2}$ [5], [23]. The consistency of each choice of σ_t was checked with $\chi^2 = \sum w_i (g_i - g_w)^2$, which should be close to the number of degrees of freedom (11) if the uncertainties are realistic.

3.4. Systematic Timing Offset

If every measured time t_m differs from the true fall time by a constant offset δ (negative if the watch is stopped early or started late), then

$$g_{\text{exp}} = \frac{2h}{t_m^2} = g \left(1 - \frac{\delta}{t_m}\right)^2 \approx g \left(1 - \frac{2\delta}{t_m}\right) \quad (8)$$

so a negative offset inflates g_{exp} , and does so most strongly for short falls. Equivalently, $t_m = \sqrt{(2h)/g} + \delta$ is linear in $\sqrt{(2h)}$, and a straight-line fit gives g from the slope and δ from the intercept.

4. RESULTS

Table 2 lists the results for all configurations, and Fig. 1 shows g_{exp} against the mean fall time.

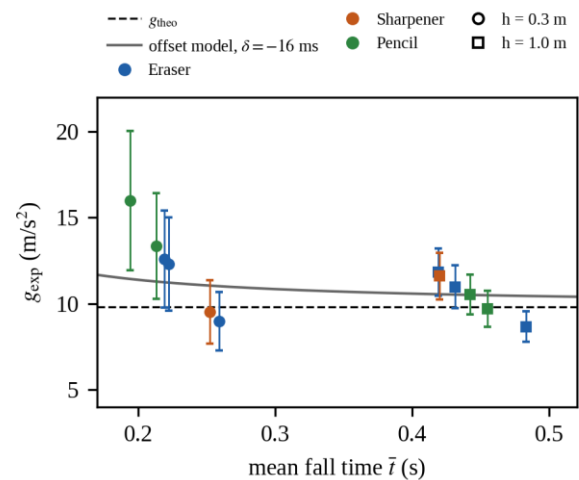


Fig. 1. Experimental g against mean fall time. Error bars: σ_g from Eq. (7) with the empirical $\sigma_t = 24.5$ ms. Dashed line: g_{theo} . Grey curve: Eq. (8) with $g = g_{\text{theo}}$ and a constant offset $\delta = -16$ ms; it captures the rise of g_{exp} at short fall times.

4.1. Air Drag

Including drag raises g_{exp} by 0.05% (PT02) to 1.45% (ET06), with a mean of 0.37% (Table 2, column Δ_{drag}). The approximation $kh/3$ of Eq. (5) reproduces every value to

within 0.02 percentage points; for example, ET06 has $kh/3 = 0.0431 \times 1.0/3 = 1.44\%$. The correction is largest for the eraser on its largest face at 1.0 m, as expected, since $k \propto A/m$ and the effect grows with h .

Table 2: Results for all 12 configurations (N = 4 trials each)

Run	Object, face	h (m)	$\bar{t}$ (s)	g_{exp} (m/s ²)	S (m/s ²)	SE (m/s ²)	σ_g (50 / 24.5 ms) (m/s ²)	δg_{rel} (%)	Δ_{drag} (%)	a_{air} (m/s ²)
ET01	Eraser, smallest	0.3	0.259	8.98	0.48	0.24	3.47 / 1.70	−8.4	0.11	0.058
ET02	Eraser, medium	0.3	0.219	12.58	0.85	0.43	5.74 / 2.81	+28.3	0.22	0.161
ET03	Eraser, largest	0.3	0.222	12.30	0.72	0.36	5.54 / 2.71	+25.5	0.43	0.312
PST01	Sharpener, planar	0.3	0.252	9.51	0.59	0.30	3.77 / 1.85	−3.0	0.15	0.082
PT01	Pencil, largest	0.3	0.213	13.35	0.78	0.39	6.27 / 3.07	+36.2	0.08	0.065
PT02	Pencil, smallest	0.3	0.194	15.98	1.12	0.56	8.24 / 4.04	+63.0	0.05	0.044
ET04	Eraser, smallest	1.0	0.483	8.66	0.25	0.13	1.79 / 0.88	−11.6	0.36	0.183
ET05	Eraser, medium	1.0	0.431	10.97	0.34	0.17	2.55 / 1.25	+11.9	0.72	0.458
ET06	Eraser, largest	1.0	0.419	11.83	0.36	0.18	2.82 / 1.38	+20.7	1.45	0.955
PST02	Sharpener, planar	1.0	0.420	11.60	0.38	0.19	2.76 / 1.35	+18.3	0.48	0.326
PT03	Pencil, largest	1.0	0.442	10.52	0.28	0.14	2.38 / 1.17	+7.3	0.28	0.168
PT04	Pencil, smallest	1.0	0.455	9.70	0.31	0.16	2.13 / 1.04	−1.0	0.15	0.089

$\bar{t}$: mean fall time; g_{exp} : drag-corrected mean of the trial values; S , SE : sample standard deviation and standard error; σ_g : timing uncertainty from Eq. (7) with the nominal $\sigma_t = 50$ ms and with the empirical $\sigma_t = 24.5$ ms; δg_{rel} : signed deviation from $g_{\text{theo}} = 9.802$ m/s²; Δ_{drag} : change in g due to including drag (Eq. 5); a_{air} : drag deceleration at impact, kv^2 .

The drag deceleration at impact, $a_{\text{air}} = kv^2$, reaches 0.96 m/s² (about 10% of g) for ET06, but this is its value at the end of the fall. Averaged over the fall, drag is much weaker, which is why the effect on g is only 1.45%. The impact value should therefore not be used as a measure of the error caused by neglecting drag.

Because $\Delta_{\text{drag}} \propto k \propto C_d/m$, a 10% error in either C_d or m changes g_{exp} by at most $0.1 \times 1.45\% \approx 0.14\%$. Using nominal masses and literature drag coefficients is therefore entirely adequate here, and every drag correction is more than ten times smaller than the timing uncertainty of the same configuration.

4.2. Timing: Repeatability versus Accuracy

The within-set standard deviations S correspond to a timing repeatability of $S/\bar{t}(2g_{\text{exp}}) \approx 6\text{--}8$ ms for every configuration. However, configurations dropped from the same height should have nearly identical fall times—drag differences change them by less than 0.7%—yet the measured means at 0.3 m range from 0.194 to 0.259 s, and those at 1.0 m from 0.419 to 0.483 s. After subtracting the expected free-fall time at each height, the residuals have an RMS scatter of 24.5 ms at both heights.

Hand timing therefore repeats well within a set of trials but varies between sets by several times more. The operator's timing habits are evidently not constant from one set to the next, and this configuration-to-configuration variation, rather than the trial-to-trial scatter, is what limits the accuracy of each result. For this reason SE values of 0.13–0.56 m/s² (Table 2) greatly understate the true uncertainty of g_{exp} .

The nominal $\sigma_t = 50$ ms has the opposite problem. With it, $\chi^2 = 2.7$ for 11 degrees of freedom, meaning the scatter of

the results is much smaller than those error bars imply. With the empirical $\sigma_t = 24.5$ ms, $\chi^2 = 11.2$ for 11 degrees of freedom, as expected for realistic uncertainties. We therefore adopt $\sigma_t = 24.5$ ms.

4.3. Combined Estimate of g

The inverse-variance weighted mean of the twelve configurations is $g_w = 10.29 \pm 0.42$ m/s², which is 5.0% above g_{theo} , a difference of 1.2 standard uncertainties. The unweighted mean, 11.33 m/s², is higher because it gives equal weight to the imprecise short drops. Computing g from the mean fall time at each height gives 11.70 ± 1.04 m/s² at 0.3 m and 10.25 ± 0.46 m/s² at 1.0 m.

For comparison with the original analysis, the mean absolute relative deviation of the twelve configurations is 19.6%. This figure hides the direction of the errors, however: eight of the twelve results lie above g_{theo} , and the signed mean deviation is +15.6% (+23.6% at 0.3 m, +7.6% at 1.0 m). The single configuration closest to g_{theo} , PT04 (−1.0%), should not be interpreted as an optimal setup: PT03, the same pencil at the same height in a different orientation, deviates by +7.3%, and the drag corrections of the two differ by only 0.13 percentage points. The difference between them is timing scatter.

4.4. Evidence for a Systematic Early Stop

The positive bias, and its growth at short fall times, points to a negative timing offset (Eq. 8). Fixing $g = g_{\text{theo}}$, the mean difference between measured and expected fall times is $\delta = -16 \pm 7$ ms (−21 ms at 0.3 m and −10 ms at 1.0 m; these two values agree within their uncertainties). As Fig. 1 shows, this single offset reproduces the trend of g_{exp} with fall time. A likely cause is anticipation: the operator sees the object

approaching the floor and stops the watch slightly before impact, while the start, tied to the operator's own release, is comparatively prompt.

Fitting $t_m = \sqrt{(2h)/g} + \delta$ with both parameters free (Fig. 2) gives $g = 8.8 \pm 1.2 \text{ m/s}^2$ and $\delta = -34 \pm 25 \text{ ms}$. Both are consistent with g_{theo} and with the fixed- g offset, but with only two heights the two parameters are strongly correlated (correlation coefficient -0.96) and poorly constrained. The method is sound; the data set is simply too narrow in height to exploit it. Five or more heights spanning a wider range would determine both g and δ independently.

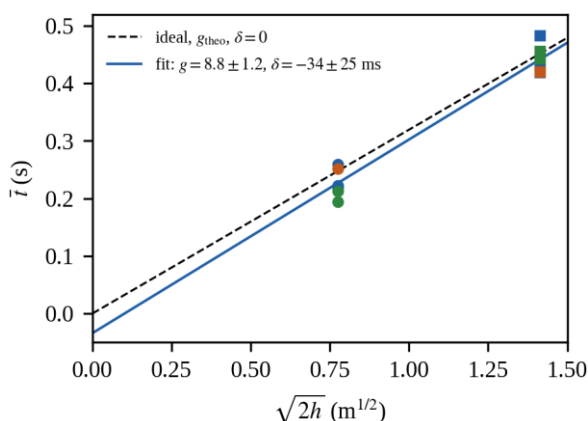


Fig. 2. Mean fall time against $\sqrt{(2h)}$. The free-fall line through the origin uses g_{theo} ; the solid line is a two-parameter fit whose slope gives g and whose intercept gives the timing offset δ . Colours and symbols as in Fig. 1.

5. DISCUSSION

The results give a clear answer to the question posed in the introduction. For household objects falling one metre or less, air drag changes the extracted g by at most 1.5% and typically by a few tenths of a percent, so solving the full drag equation is unnecessary; the estimate $\Delta_{\text{drag}} \approx kh/3$ tells a student in one line whether drag can be ignored. Drag becomes important only for objects with a large area-to-mass ratio (paper, feathers, balloons) or for much longer falls [4], [14].

Timing is the dominant source of error, but not in the way usually assumed. Treating reaction time as a large random uncertainty ($\pm 50 \text{ ms}$) overstates the scatter of the results, while the within-set standard error understates the error of each result. What the data show instead is a moderate between-set variation of about 25 ms together with a systematic early stop of about 16 ms. The systematic component matters because it does not average away with more trials; it biases g upwards, most severely for short drops (Eq. 8). This explains why the 0.3 m results overestimate g by 24% on average and why raising the drop height reduces the bias.

These findings suggest simple improvements to the classroom experiment:

- Drop from several heights (at least five, e.g. 0.5–2.5 m) and fit t against $\sqrt{(2h)}$. The intercept measures the operator's offset, and the slope gives g free of it.

- Record some drops with a phone's slow-motion video (240 fps, 4 ms per frame) [6] and compare them with the stopwatch times to measure the operator's offset directly.
- Report the spread between configurations, not only within them, and quote a weighted mean with its uncertainty rather than a mean absolute deviation.

Limitations. Only two heights were used, so g and δ could not be determined independently. Object masses were nominal, not measured, although Section 4.1 shows this does not affect the conclusions. All timing was done by a single operator, so the size of the offset may not generalise. Tumbling and small rotations during the fall were monitored only visually.

6. CONCLUSION AND FUTURE WORK

Hand-timed drops of an eraser, a pencil sharpener and a pencil from 0.3 and 1.0 m give $g = 10.29 \pm 0.42 \text{ m/s}^2$, consistent within 1.2 standard uncertainties with the local value 9.802 m/s^2 . Air drag contributes at most 1.45% (mean 0.37%) and is well predicted by $kh/3$. The accuracy of the method is limited instead by timing: between-set variation of about 25 ms and a systematic early stop of about 16 ms, which together inflate g most strongly for short drops. Future work will extend the experiment to five or more heights up to about 3 m, calibrate the operator's offset against slow-motion video, weigh the objects, and include a high-drag object such as a paper cone, for which the drag correction becomes large enough to test the model directly.

DECLARATIONS

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Author Contributions

E. Çiçek conceived the study, performed the experiments, analysed the data and wrote the manuscript.

Data Availability Statement

The individual trial times and the analysis script are available from the corresponding author upon reasonable request.

Conflict of Interest

The author declares that there are no conflicts of interest.

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Ethics Approval and Consent to Participate

Not applicable. This study involves standard educational physics measurements with non-hazardous everyday objects; no human subjects other than the author (as timer), personal data or animals were involved.



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